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# AdS Twistors for Higher Spin Theory

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**Abstract:** We construct spectra of supersymmetric higher spin theories in  $D = 4, 5$  and 7 from twistors describing massless (super-)particles on AdS spaces. A massless twistor transform is derived in a geometric way from classical kinematics. Relaxing the spin-shell constraints on twistor space gives an infinite tower of massless states of a “higher spin particle”, generalising previous work of Bando *et al.*. This can generically be done in a number of ways, each defining the states of a distinct higher spin theory, and the method provides a systematic way of finding these. We reproduce known results in  $D = 4$ , minimal supersymmetric 5- and 7-dimensional models, as well as supersymmetrisations of Vasiliev’s Sp-models as special cases. In the latter models a dimensional enhancement takes place, meaning that the theory lives on a space of higher dimension than the original AdS space, and becomes a theory of doubletons. This talk was presented at the XIXth Max Born Symposium “Fundamental Interactions and Twistor-Like Methods”, September 2004, in Wrocław, Poland.

## 1. INTRODUCTION

It has been known for some time that higher spin theory, *i.e.*, theories of interacting massless higher spin fields, should be formulated in anti-de Sitter space, or more precisely, allow AdS space as a vacuum solution [1]. The theory of higher spin fields was subsequently developed in a series of papers [2]. For excellent recent reviews, see refs. [3,4,5], which also give an account to the earlier history of higher spin.

In recent years, there has been a growing interest in the theory of massless higher spins. There is some hope that such a theory may provide a geometric framework and a symmetry principle underlying string theory, which in that case would be interpreted as a broken phase of higher spin theory, where the higher spin fields have become massive. For the possible connection between string theory and higher spin theory, see refs. [6,7,8,9,10].

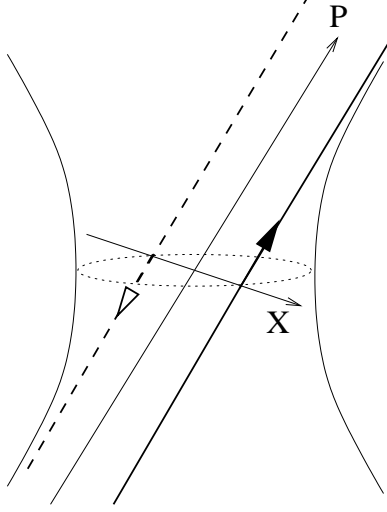
Higher spin theory has mostly been constructed using spinorial oscillators. These are the kinds of models that will be discussed in the present talk. Recently, constructions with vectorial variables, for any dimension, have been performed [11], and we will have nothing to say about these. The purpose of the work presented in this talk is to present a unified framework for obtaining spinorial (twistorial) variables for higher spin theory by relaxation of spin-shell constraints for ordinary bosonic or supersymmetric particles. The discussion is performed entirely at the kinematic, non-interacting, level.

## 2. TWISTOR TRANSFORM FOR MASSLESS PARTICLES ON ADS

Consider  $\text{AdS}_{d+1}$  space with radius  $R$  as the hyperboloid

$$x_M x^M \equiv -(x^0)^2 - (x^{0'})^2 + \sum_{i=1}^d (x^i)^2 = -R^2 \quad (2.1)$$

in flat space with signature  $(2, d)$ . The trajectory of a massless particle, a light-like geodesic, is the intersection of the hyperboloid with a plane through the origin spanned by one light-like and one time-like vector, *i.e.*,  $x = \alpha X + \beta P$ , where  $P^2 = 0$ ,  $X^2 < 0$  and  $X \cdot P = 0$ .  $X$  may then be seen as the coordinate for the location of the particle, fulfilling eq. (2.1), and  $P$  as its momentum, being light-like and directed along the hyperboloid. A plane is also defined by a bi-vector  $\Pi^{MN}$  which is *simple*, meaning that it can be expressed in terms of two vectors as  $\Pi^{MN} = X^{[M} P^{N]}$ .



*Fig. 1. A light-like geodesic as the intersection of a plane and a hyperboloid.*

The condition that  $\Pi$  is simple can be expressed as

$$\Pi^{[MN}\Pi^{PQ]} = 0 . \quad (2.2)$$

The properties of the vectors  $X$  and  $P$  imply in addition that

$$\Pi_{MN}\Pi^{MN} = 0 . \quad (2.3)$$

We want to find a twistor transform for a massless particle in AdS space. This means finding an expression  $\Pi$  fulfilling eqs. (2.2) and (2.3) as a spinor bilinear modulo some well defined constraints and transformations. The spinor should be a spinor under the  $\text{AdS}_{d+1}$  group  $\text{Spin}(2, d)$ . We restrict ourselves to the cases  $d + 1 = 4, 5, 7$ , which are naturally related to the division algebras  $\mathbb{K}_1 = \mathbb{R}$ ,  $\mathbb{K}_2 = \mathbb{C}$  and  $\mathbb{K}_4 = \mathbb{H}$ . Using the isomorphisms  $Sp(4; \mathbb{K}_\nu) \approx \text{Spin}(2, \nu + 2)$ , the spinors are 4-component with elements in  $\mathbb{K}_\nu$ .

The form of the twistor transform is

$$\Pi^{MN} = \frac{1}{8} \Lambda^\dagger \Gamma^{MN} \Lambda , \quad (2.4)$$

where  $I$  is an internal index that will turn out to run over two values, or equivalently, since the adjoint of  $\text{Spin}(2, \nu + 2)$  can be represented as a hermitean  $4 \times 4$ -matrix,  $\Pi^{A\dot{B}} = \frac{1}{2}\Lambda^{AI}\Lambda^{\dagger I\dot{B}}$ , or simply

$$\Pi = \frac{1}{2}\Lambda\Lambda^{\dagger} . \quad (2.5)$$

$\Pi$  contains the generator of AdS transformations, if  $\Lambda$  has canonical Poisson brackets with itself.

It is not possible to form a simple bivector from a single AdS spinor, the minimum number is two. That this is true can be checked explicitly for some simple specific choice of the plane. Take  $\Pi^{\oplus 0} \neq 0$  (with the light-like  $\oplus$  direction and the time-like 0 direction orthogonal) and the rest of the components vanishing. Using the gamma matrices of the Appendix, and denoting the spinor

$$\Lambda = \begin{bmatrix} \lambda^{\alpha} \\ \mu_{\dot{\alpha}} \end{bmatrix} = \begin{bmatrix} k \\ l \\ m \\ n \end{bmatrix} \quad (2.6)$$

(suppressing the index  $I$ ), one needs

$$\begin{aligned} \Lambda^{\dagger}\Gamma^{\oplus 0}\Lambda &\sim \bar{k}k + \bar{l}l \neq 0 \\ \Lambda^{\dagger}\Gamma^{\oplus, \nu+1}\Lambda &\sim \bar{k}k - \bar{l}l = 0 \\ \Lambda^{\dagger}\Gamma^{\oplus i}\Lambda &\sim \bar{k}\bar{e}_i l + \bar{l}e_i k = 0 , \quad i = 1, \dots, \nu \\ \Lambda^{\dagger}\Gamma^{\ominus 0}\Lambda &\sim \bar{m}m + \bar{n}n = 0 \\ \Lambda^{\dagger}\Gamma^{\ominus, \nu+1}\Lambda &\sim \bar{m}m - \bar{n}n = 0 \\ \Lambda^{\dagger}\Gamma^{\ominus i}\Lambda &\sim \bar{m}\bar{e}_i n + \bar{n}e_i m = 0 , \quad i = 1, \dots, \nu \end{aligned} \quad (2.7)$$

In order to satisfy the first three equations, two spinors are needed. Then the last three equations imply that  $\mu = 0$ . The rest of the components of  $\Pi$  mix  $\lambda$  and  $\mu$  and will thus vanish.

A pair of spinors has an “R-symmetry”, acting from the right on the twistor  $\Lambda \in \mathbb{K}_{\nu}^8$  of the form

$$\Lambda \sim \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} , \quad (2.8)$$

generated by anti-hermitean  $2 \times 2$ -matrices with entries in  $\mathbb{K}_\nu$ . The corresponding groups are<sup>\*</sup>

$$A_2(\mathbb{K}_\nu) = \begin{cases} \text{U}(1) \\ \text{SU}(2) \times \text{U}(1) \\ \text{Spin}(5) \end{cases} \quad (2.9)$$

Note that although these symmetries are “internal”, and commute with the AdS generators, they are isomorphic to the groups of transverse rotations for a massless particle (except for the extra  $\text{U}(1)$ , whose rôle is commented on below). Denote the corresponding generators  $T$ . The specific spinor  $\Lambda$  above clearly has  $T = 0$ , since all generators contain “ $\mu\lambda$ ”. The rest of the conditions turn out to be consequences of the choice of frame. It is also clear that since  $\Pi$  commutes with  $T$ , spinors should be counted modulo  $T$ -transformations. In fact,

$$T = \frac{1}{2}\Lambda^\dagger\Lambda \approx 0 \quad (2.10)$$

is the gauge invariance of the twistor. The generators  $T$  are anti-hermitean due to the anti-hermiticity of the “spinor metric”  $\epsilon$  (see the Appendix). The counting of physical degrees of freedom is straightforward:

$$2 \times 4\nu - 2(3\nu - 2) = 2\nu + 4 = 2d, \quad (2.11)$$

which is the correct number for the phase space of a massless particle in  $d + 1$  dimensions.

A twistor construction for massive particles was performed in refs. [17,18,19]. It was noted that four spinors were needed to form a plane whose intersection with the hyperboloid is a massive geodesic for  $\text{AdS}_4$  and  $\text{AdS}_7$ , while two spinors were sufficient for  $\text{AdS}_5$ . In the last of these cases, the  $\text{U}(1)$  generator was identified with the mass, and the  $\text{SU}(2)$  were gauge generators. In the 4- and 7-dimensional cases, it was necessary to break the “R-symmetry”  $A_4(\mathbb{K}_\nu)$  by identifying a non-vanishing  $\text{U}(1)$  generator as the mass, which lead to a mixture of first and second class constraints. The present case of massless twistors is more uniform in the different dimensionalities, and only first class constraints are present.

We should stress a difference between twistor transforms on AdS space and Minkowski space. The division algebra construction [20,14] is natural in Minkowski space of dimensions  $\nu + 2 = 3, 4, 6$  and  $10$ . The basic relation is

$$p \sim \lambda\lambda^\dagger, \quad (2.12)$$

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<sup>\*</sup> Here it becomes clear why  $\text{AdS}_{11}$  is not included, although one has the isomorphism  $\text{Sp}(4;\mathbb{O}) \approx \text{Spin}(2,10)$ . “ $A_2(\mathbb{O})$ ” does not form a Lie algebra, not even in the soft sense that  $A_1(\mathbb{O}) \approx S^7$  does [12,13,14,15,16]; it forms a set of generators in a  $\text{Spin}(9)/G_2$  coset of  $\text{Spin}(9)$ .

which due to Fierz identities in these dimensionalities ensure that  $p$  is light-like. Such a construction is not possible in AdS, where the isometry group is semi-simple and does not contain translations. Any natural twistor transform will involve the whole twistor phase space, as in eq. (2.5), not only a commuting half of it as in eq. (2.12). Also in Minkowski space, however, eq. (2) can be used as a definition of the twistor transform, when the spinor  $\lambda^\alpha$  is complemented with its canonical conjugate  $\mu_{\dot{\alpha}}$  and  $\Pi$  is interpreted as generators of conformal transformations. This interpretation is relevant also in AdS space, where eq. (2.12) is seen as the twistor transform on the boundary, involving a commuting half of the AdS twistor, which will be utilised below.

We finally note that the division algebra language is well suited to display certain Fierz identities relating the behavior under AdS transformations and under the “R-symmetry”. We note that a  $4 \times 4$   $\mathbb{K}_\nu$ -valued matrix carrying two spinor indices  $A$  and  $\dot{B}$  decomposes into three different irreducible representations: hermitean (the adjoint, and in the complex case a singlet), anti-hermitean traceless (a 4-form, which in the real case is a vector, in the complex case the adjoint and in the quaternionic case self-dual) and the trace (singlet). If we form the twistor quadrilinear

$$\text{III} = \Lambda \Lambda^\dagger \Lambda \Lambda^\dagger, \quad (2.13)$$

it will be anti-hermitean and vanish due to  $T = 0$ . The vanishing of the 4-form part shows that  $\Pi$  is simple as in eq. (2.2), and the singlet that  $\Pi_{MN} \Pi^{MN} = 0$ . When we later relax the constraint  $T = 0$  to obtain “higher spin particles”, such Fierz identities will relate Casimirs of the AdS group to Casimirs of R-symmetry.

The construction is straightforwardly extended to superparticles by introducing  $2N$  anticommuting scalar variables  $\Theta$  arranged in an  $N \times 2$  matrix, transforming from the left by  $O(N; \mathbb{K}_\nu)$  and from the right by  $A_2(\mathbb{K}_\nu)$ . The supertwistor

$$\Xi = \begin{bmatrix} \Lambda \\ \Theta \end{bmatrix} \quad (2.14)$$

transforms in the fundamental of the  $N$ -extended  $\text{AdS}_{\nu+3}$  supergroup  $OSp(N|4; \mathbb{K}_\nu)$ . The constraint structure is not affected, we now have  $T = \frac{1}{2} \Xi^\dagger \Xi = \frac{1}{2} (\Lambda^\dagger \Lambda + \Theta^\dagger \Theta) \approx 0$ , still generating  $A_2(\mathbb{K}_\nu)$ .

### 3. TWISTORS FOR HIGHER SPIN

We will now relax the constraint  $T = 0$  in order to incorporate higher spin massless states in the model. Since four spinors are needed in the 4- and 7-dimensional models to describe

massive states, it is essentially clear that such states will not appear. In a twistor model, the on-shell constraint is not a mass-shell constraint, but rather a spin-shell constraint, so relaxing it will lead to a multitude of spins. In the 5-dimensional case we need to be careful—since the  $U(1)$  generator measures the mass we are only to relax the  $SU(2)$  constraint (which matches nicely with the observation that this is isomorphic to transverse rotations for a massless particle in five dimensions).

It is possible to relax only part of the spin-shell constraints. In  $AdS_4$  with gauge group  $U(1)$ , there is no choice, but in the other cases keeping as gauge group any subgroup of  $SU(2)$  or  $Spin(5)$  (smaller than the group itself) should define a distinct higher spin theory. Since the generators  $T$  are AdS scalars, this procedure is covariant, and will result in restrictions on the massless representations occurring. In this sense, the “smallest” higher spin theory should be given by the “biggest” subgroup. For example, one could consider a higher spin theory on  $AdS_5$  with internal “spin manifold” (the non-gauged part of the R-symmetry)  $SU(2)/U(1) = S^2$  or a theory on  $AdS_7$  with spin manifold  $Spin(5)/(SU(2) \times SU(2))$ . These choices give the minimal models considered by Sezgin and Sundell [21,22]. If, on the other hand, the subgroup chosen is the trivial one, one obtains the “Sp space-times” of ref. [23].

Quantisation is straightforward and goes as follows. States in unitary representations of the AdS group are formed by letting an (anti-)commuting subset of variables in  $\Xi$  (a configuration space) act on a (preferably scalar) vacuum. States are thus obtained as polynomials in half of the supertwistor variables. For an ordinary superparticle, where  $T \approx 0$ , the constraints are implemented by only considering states that are R-symmetry singlets. If a subset of generators are kept as gauge generators, as in the minimal models, they are treated accordingly. A natural choice of configuration space for the bosonic variables is the upper half  $\lambda$  of  $\Lambda$ , which is a pair of spinors under the Lorentz group acting on the boundary of AdS space.

The minimal models in 4, 5 and 7 dimensions have gauge groups  $\emptyset$ ,  $U(1) \times U(1)$  and  $SU(2) \times SU(2)$ , respectively. The two factors act each on one column of  $\Xi$  and do not mix them—the two spinors decouple with this choice of spin constraints. The states of the minimal theories are thus obtained as the tensor product of two representations obtained from a single AdS spinorial oscillator, *i.e.*, as the tensor product of two singletons/doubletons, as in refs. [21,22].

Let us for a moment dwell on the “maximal” models, the supersymmetric versions of the Sp-models. It is trivial to write an action for the higher spin particle,

$$S = \int d\tau \Xi^\dagger \dot{\Xi} . \quad (3.1)$$

(for  $\text{AdS}_5$  we should still include a Lagrange multiplier for the mass  $\text{U}(1)$ ). From just inspecting the action, we see that the higher spin particle is invariant under a much larger symmetry than the AdS group. In 4 and 7 dimensions, where there are no constraints, this is the real orthosymplectic supergroup acting on the number of real fermionic and bosonic components of the supertwistor, *i.e.*,  $\text{OSp}(2N|8)$  (as noted in ref. [24]) and  $\text{OSp}(8N|32)$ , respectively. In  $\text{AdS}_5$  we have the subgroup of  $\text{OSp}(4N|16)$  that commutes with  $\text{U}(1)$ , which is  $\text{SU}(2N|4,4)$ . The corresponding groups acting on the (bosonic part of) configuration space, and thus extending the Lorentz symmetry on the boundary, are  $\text{SL}(4)$ ,  $\text{SL}(4, \mathbb{C})$  and  $\text{SL}(16)$ .

As mentioned earlier, the twistor parametrisation of the AdS generators  $\Pi$  imply a direct relation between Casimirs of the AdS group and of the internal symmetry. This relation follows immediately from equations like (2.13) and reads

$$\text{tr}\Pi^n = \text{tr}T^n . \quad (3.2)$$

One might worry that once the spin-shell constraint is relaxed, the relation to space-time is lost. The following consideration shows that this is not the case. A simple bi-vector, describing a plane, can be written as  $\Pi^{MN} = X^{[M}P^{N]}$ , where  $X^2 = -R^2$ ,  $X \cdot P = 0$  ( $P$  is tangent to the hyperboloid) and  $P^2 = 0$ . When  $T \neq 0$ , the best one can do is  $\Pi^{MN} = X^{[M}P^{N]} + S^{MN}$ . In order to still have  $\text{tr}\Pi^2 = \text{tr}T^2$ , one should demand that  $X_M S^{MN} = 0$  (spin is in the AdS tangent space) and  $P_M S^{MN} = 0$  (spin is transverse). In addition,  $S$  is defined modulo  $\delta S^{MN} = V^{[M}P^{N]}$ , with  $V_M X^M = V_M P^M = 0$ , which can be absorbed in a redefinition of  $X$  (a reflection of gauge invariance). Such an  $S$  is restricted to lie in the transverse rotations, and one will have  $\text{tr}\Pi^n = \text{tr}S^n$ . This also shows that although  $T$  are the generators of an algebra of *internal* rotations, the parametrisation of the AdS generators in terms of spinors (the twistor transform) implies the identification of all spin quantum numbers with quantum numbers with respect to  $T$ , and the isomorphism between the internal algebra and the algebra of transverse spin degrees of freedom becomes a physical identification.

In the spirit of ref. [23], we can ask questions about the causal structure in such a theory. Vasiliev showed in ref. [23] that the causal structure is determined by the maximal Clifford algebras contained in the representations of the bilinears in  $\lambda$ . They must contain matrices acting on 4-component real, 4-component complex and 16-component real spinors, respectively, considering that the subalgebras acting linearly on the configuration space of boundary spinors are  $\text{SL}(4)$ ,  $\text{SL}(4, \mathbb{C})$  and  $\text{SL}(16)$ . We note that these groups are large enough to contain as subgroups the Lorentz groups in higher dimensions, and in fact, since we consider pairs of division algebra spinors, the dimensions corresponding to choosing the next larger division algebra. This means that the boundary in the maximal higher spin model



in dimensions 4, 5 and 7 may be considered as 4-, 6- and 10-dimensional. Interpreted in the higher dimensionalities, the pair of spinors becomes a single spinor, and the corresponding states are those of a doubleton in 5, 7 and 11 dimensions.

The maximal dimensionality of the Minkowski space is then 4, 6 and 10, respectively, where the boundary spinors are 2-component complex, quaternionic and octonionic. When we thus consider the  $SL(2, \mathbb{K}_{2\nu}) \approx Spin(1, 2\nu + 1)$  subgroup of the boundary group, the bilinears in  $\lambda$  decompose into a vector and a set of tensorial charges. In the theory originating from  $AdS_4$ , where the boundary has now become 4-dimensional, we have the vectorial coordinates and a 2-form (this is the “tensorial” model considered by Bandos *et al.* [24,25,26,27]). In the theory on  $AdS_5$ , where the boundary is enhanced to be 6-dimensional, we have the vector and an  $SU(2)$ -triplet of self-dual 3-forms, and in the  $AdS_7$  model, with a 10-dimensional boundary, there is the vector and a self-dual 5-form.

Starting out from a theory in  $AdS_{d+1}$  space, with  $d = 4, 5, 7$ , we end up with a different space-time interpretation, where the “coordinates” are the ordinary coordinates together with a set of forms, “central charge coordinates”. The latter provide an alternative picture of the spin degrees of freedom in higher spin theory. However, the general picture is more intricate than in 4 dimensions, where the alternative space-time also is 4-dimensional. We see that the alternative descriptions of the 5- and 7-dimensional AdS theories are theories in 6- and 10-dimensional Minkowski space.

Some more things can be made explicit about the relation between the two descriptions/interpretations. Let us take the  $AdS_7$  theory as an example. The original AdS group is  $Spin(2, 6) \approx Sp(4; \mathbb{H})$ , and the internal symmetry group is  $Spin(5) \approx A_2(\mathbb{H})$ .  $Sp(4; \mathbb{H}) \times A_2(\mathbb{H})$  is a subgroup of the group of all symplectic transformations on the bispinor, which is  $Sp(32)$ . Restricting to transformations on configuration space, *i.e.*, on half the spinor, breaks  $Sp(4; \mathbb{H}) \times A_2(\mathbb{H})$  to  $SL(2; \mathbb{H}) \times A_2(\mathbb{H}) \approx Spin(1, 5) \times Spin(5)$  and  $Sp(32)$  to  $SL(16)$ . When we looked for the maximal Clifford algebra above, that procedure singled out  $Spin(1, 9) \approx SL(2; \mathbb{O})$ . The  $Spin(1, 5)$  group is smaller than the Lorentz group on the tangent space of  $AdS_7$ . It can be identified with the Lorentz group on the conformally Minkowski boundary of  $AdS_7$ . This gives a picture of what happens: the boundary Lorentz group is a common subgroup of the AdS group and the boundary symmetry group (in this case  $Sp(16)$ ). While adjoining a radial coordinate gives the AdS picture, the alternative picture is obtained by supplementing the boundary coordinates with a number of “spin degrees of freedom”, manifested as tensorial variables. While this discussion refers to the configuration space, or the boundary variables, one may equally well consider the dimensionally enhanced model as a bulk theory on  $AdS_{11}$ . There the states are those obtained from a single supertwistor, which means that it is a doubleton in the sense of ref. [28,29].

#### 4. CONCLUDING REMARKS

We have given a systematic way of deriving different versions of bosonic and supersymmetric higher spin theory on anti-de Sitter spaces of dimension 4, 5 and 7, *i.e.*, the dimensionalities related to the real, complex and quaternionic division algebras. The construction, as it stands is first-quantised, and can be understood as describing the dynamics of a (free) higher spin particle.

The main message is that starting with a twistorial description of particle mechanics, the spin-shell constraints may be relaxed in a systematic and controlled way to yield higher spin degrees of freedom. Using this prescription produces all known spinorial descriptions of higher spin kinematics as special cases. So far, nothing really new has come out of the present work, although we think it provides a nice and unified framework for known models, and a better understanding of their respective rôles and relations.

It would be interesting, especially considering the potential relation of higher spin theory to string theory [6,7,8,9,10], to see if one can use the particle action and couple it to background fields describing exactly the states produced by the first-quantised action itself, and what information about the interacting theory may be obtained this way.

#### APPENDIX A: SPINORS AND GAMMA MATRICES

Here, we set the conventions for the gamma matrices used in the twistor transform (2.4). They can be given in a unified notation for the three cases. We denote by  $e_i$ ,  $i = 1, \dots, \nu$ , the standard orthonormal basis for the division algebra  $\mathbb{K}_\nu$ . For the flat embedding space with signature  $(2, \nu + 2)$  we use light-cone coordinates  $M = (\oplus, \ominus, \mu) = (\oplus, \ominus, +, -, i)$  and scalar product  $V \cdot W = -V^\oplus W^\ominus - V^\ominus W^\oplus - V^+ W^- - V^- W^+ + V^i W^i$ .

A spinor under the AdS group belongs to the fundamental representation of  $\text{Sp}(4; \mathbb{K}_\nu)$ , *i.e.*, it is a 4-component column with entries in  $\mathbb{K}_\nu$ . We use a dotted/undotted notation for spinors, and in addition primed and unprimed spinor indices (since there generically are two chiralities). The two spinor representations, with indices  $A$  and  $A'$ , both decompose into as  $A \rightarrow (\alpha, \dot{\alpha}) \leftarrow A'$  under the subgroup  $\text{SL}(2; \mathbb{K}_\nu) \approx \text{Spin}(1, \nu + 1)$ . The gamma matrices (or, strictly speaking, sigma matrices) acting on one chirality (the unprimed one that is chosen for the twistors) are

$$\begin{aligned} \Gamma^{\oplus A'}_B &= \begin{bmatrix} \sqrt{2} \mathbb{1}^{\alpha}_{\beta} & 0 \\ 0 & 0 \end{bmatrix}, & \Gamma^{\ominus A'}_B &= \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{2} \mathbb{1}_{\dot{\alpha} \dot{\beta}} \end{bmatrix}, \\ \Gamma^{\mu A'}_B &= \begin{bmatrix} 0 & \tilde{\gamma}^{\mu \alpha \dot{\beta}} \\ \gamma^{\mu}_{\dot{\alpha} \beta} & 0 \end{bmatrix}, \end{aligned} \tag{A.1}$$

and on the other one

$$\begin{aligned}\tilde{\Gamma}^{\oplus A}{}_{B'} &= \begin{bmatrix} 0 & 0 \\ 0 & -\sqrt{2}\mathbb{1}_{\dot{\alpha}\dot{\beta}} \end{bmatrix}, & \tilde{\Gamma}^{\ominus A}{}_{B'} &= \begin{bmatrix} -\sqrt{2}\mathbb{1}^{\alpha\beta} & 0 \\ 0 & 0 \end{bmatrix}, \\ \tilde{\Gamma}^{\mu A}{}_{B'} &= \begin{bmatrix} 0 & \tilde{\gamma}^{\mu\alpha\dot{\beta}} \\ \gamma^{\mu}{}_{\dot{\alpha}\beta} & 0 \end{bmatrix},\end{aligned}\tag{A.2}$$

where  $\gamma^{\mu}, \tilde{\gamma}^{\mu}$  are  $\text{SL}(2; \mathbb{K}_{\nu}) \approx \text{Spin}(1, \nu + 1)$  gamma matrices:

$$\begin{aligned}\gamma^{+}{}_{\dot{\alpha}\beta} &= \begin{bmatrix} \sqrt{2} & 0 \\ 0 & 0 \end{bmatrix}, & \gamma^{-}{}_{\dot{\alpha}\beta} &= \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{2} \end{bmatrix}, & \gamma^i{}_{\dot{\alpha}\beta} &= \begin{bmatrix} 0 & \bar{e}_i \\ e_i & 0 \end{bmatrix} \\ \tilde{\gamma}^{+}{}_{\alpha\dot{\beta}} &= \begin{bmatrix} 0 & 0 \\ 0 & -\sqrt{2} \end{bmatrix}, & \tilde{\gamma}^{-}{}_{\alpha\dot{\beta}} &= \begin{bmatrix} -\sqrt{2} & 0 \\ 0 & 0 \end{bmatrix}, & \tilde{\gamma}^{i\alpha\dot{\beta}} &= \begin{bmatrix} 0 & \bar{e}_i \\ e_i & 0 \end{bmatrix}.\end{aligned}\tag{A.3}$$

The matrices  $\Gamma^{MN}$  used in the construction of the plane defining the geodesics are constructed as

$$\Gamma^{MNA}{}_B = \frac{1}{2}(\tilde{\Gamma}^M\Gamma^N - \tilde{\Gamma}^N\Gamma^M)^A{}_B,\tag{A.4}$$

and the twistor bilinear is given by

$$\Pi^{MN} = \frac{1}{2}\Lambda^{\dagger}\Gamma^{MN}\Lambda \equiv \frac{1}{2}\Lambda_A^{\dagger}\Gamma^{MNA}{}_B\Lambda^B \equiv \frac{1}{2}\Lambda^{\dagger\dot{A}}\varepsilon_{\dot{A}B}\Gamma^{MNB}{}_C\Lambda^C,\tag{A.5}$$

where the anti-hermitean “spinor metric”

$$\varepsilon_{\dot{A}B} = \begin{bmatrix} 0 & \mathbb{1}_{\dot{\alpha}\dot{\beta}} \\ -\mathbb{1}^{\alpha\beta} & 0 \end{bmatrix}\tag{A.6}$$

is used to lower the spinor index, and where  $\dagger$  implies division algebra hermitean conjugation. In the real case, dots are of course superfluous, and there is only one chirality. The above formulæ are still correct, and primed and unprimed indices are then related via

$$E^{A'}{}_B = \begin{bmatrix} 0 & \varepsilon^{\alpha\beta} \\ \varepsilon_{\alpha\beta} & 0 \end{bmatrix}.\tag{A.7}$$

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